1. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

2. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\frac{1\text{n}(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x + 1)}, \; x > -\frac{1}{4}, \; y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)
(Total 13 marks)

3. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(Total 6 marks)

4. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \quad -\pi \le x \le \pi.$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)
(Total 13 marks)

5. The curve C has equation

$$x = 2 \sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \dfrac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{2}}$ = at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)
(Total 9 marks)

6. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(Total 5 marks)

7. The function f is defined by

$$\mathrm{f}: x \mapsto 3 + 2\mathrm{e}^x, \quad x \in \mathbb{R}.$$

(a) Evaluate $\int_0^1 f(x)\,dx$, giving your answer in terms of e.

(3)

The curve C, with equation $y = \mathrm{f}(x)$, passes through the y-axis at the point A. The tangent to C at A meets the x-axis at the point $(c, 0)$.

(b) Find the value of c.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto \frac{5x+2}{x+4}, \quad x \in \mathbb{R}, \quad x > -4.$$

(c) Find an expression for $\mathrm{g}^{-1}(x)$.

(3)

(d) Find gf(0).

(2)

(Total 12 marks)

8. The curve C with equation $y = k + \ln 2x$, where k is a constant, crosses the x-axis at the point $A\left(\frac{1}{2e}, 0\right)$.

(a) Show that $k = 1$.

(2)

(b) Show that an equation of the tangent to C at A is $y = 2ex - 1$.

(4)

(c) Complete the table below, giving your answers to 3 significant figures.

x	1	1.5	2	2.5	3
1 + ln 2x		2.10		2.61	2.79

(2)

(d) Use the trapezium rule, with four equal intervals, to estimate the value of

$$\int_1^3 (1 + \ln 2x) \, dx.$$

(4)

(Total 12 marks)

9. $$f(x) = x + \frac{e^x}{5}, \quad x \in \mathbb{R}.$$

(a) Find $f'(x)$.

(2)

The curve C, with equation $y = f(x)$, crosses the y-axis at the point A.

(b) Find an equation for the tangent to C at A.

(3)

(c) Complete the table, giving the values of $\sqrt{\left(x+\frac{e^x}{5}\right)}$ to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x+\frac{e^x}{5}\right)}$	0.45	0.91			

(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x+\frac{e^x}{5}\right)}\,dx$$

.

(4)
(Total 11 marks)

10. Given that $y = \log_a x$, $x > 0$, where a is a positive constant,

(a) (i) express x in terms of a and y,

(1)

(ii) deduce that $\ln x = y \ln a$.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{x \ln a}$.

(2)

The curve C has equation $y = \log_{10} x$, $x > 0$. The point A on C has x-coordinate 10. Using the result in part (b),

(c) find an equation for the tangent to C at A.

(4)

The tangent to C at A crosses the x-axis at the point B.

(d) Find the exact x-coordinate of B.

(2)
(Total 10 marks)

11. The curve C has equation $y = \text{f}(x)$, where

$$\text{f}(x) = 3 \ln x + \frac{1}{x}, \qquad x > 0.$$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.

(4)

(b) Show that the y-coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found.

(2)

The point Q on C has x-coordinate 1.

(c) Find an equation for the normal to C at Q.

(4)

The normal to C at Q meets C again at the point R.

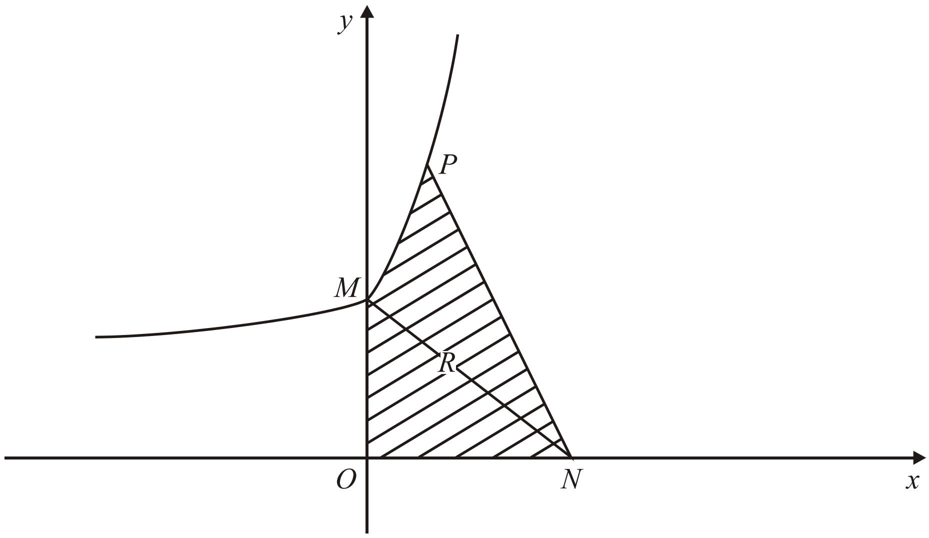
(d) Show that the x-coordinate of R

(i) satisfies the equation $6 \ln x + x + \dfrac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14.

(4)
(Total 14 marks)

12.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)
(Total 12 marks)

1. At P, $y = \underline{3}$ B1

$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$ <u>A1</u>

$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$

$m(\mathbf{N}) = \frac{-1}{-18}$ or $\frac{1}{18}$

N: $y - 3 = \frac{1}{18}(x - 2)$

N: $\underline{x - 18y + 52 = 0}$ A1

–

<u>**Note**</u>

1st $\pm k(5 - 3x)^{-3}$ can be implied. See appendix for application of the quotient rule.

2nd Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$;

3rd Uses $m(N) = -\frac{1}{\text{their } m(T)}$.

4th $y - y_1 = m(x - 2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$.

Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.

[7]

2. (i) (a) $y = x^2 \cos 3x$

Apply product rule: $\left\{\begin{array}{ll} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{array}\right\}$

Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha\, x \cos 3x \pm \beta\, x^2 \sin 3x$

$\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$ Any one term correct A1

Both terms correct and no further simplification to terms in $\cos\alpha\, x^2$ or $\sin\beta\, x^3$. A1 3

(b) $y = \dfrac{\ln(x^2+1)}{x^2+1}$

$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$ $\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$

$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$ A1

Apply quotient rule: $\left\{ \begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 2x \end{array} \right\}$

$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$ Applying $\dfrac{vu' - uv'}{v^2}$

Correct differentiation with correct bracketing but allow recovery. A1 4

$\left\{ \dfrac{dy}{dx} = \dfrac{2x - 2x\ln(x^2+1)}{(x^2+1)^2} \right\}$ {Ignore subsequent working.}

(ii) $y = \sqrt{4x+1}, x > -\frac{1}{4}$

At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$ At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$ B1

$\dfrac{dy}{dx} = \underline{\dfrac{1}{2}(4x+1)^{-\frac{1}{2}}}$ $\pm k(4x+1)^{-\frac{1}{2}}$ *

$2(4x+1)^{-\frac{1}{2}}$ A1 aef

$\dfrac{dy}{dx} = \dfrac{2}{(4x+1)^{\frac{1}{2}}}$

At P, $\dfrac{dy}{dx} = \dfrac{2}{(4(2)+1)^{\frac{1}{2}}}$ Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;

Hence m(**T**) = $\dfrac{2}{3}$

$y - y_1 = m(x - 2)$

Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or 1 $y - y_1 = m(x -$ their stated $x)$ with 'their TANGENT gradient' and

or $y = \frac{2}{3}x + c$ and their y_1; dM1 * ;

$3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$; or uses $y = mx + c$ with

	'their TANGENT gradient', their x and their y_1.		
Either **T**: $3y - 9 = 2(x - 2)$;			
T: $3y - 9 = 2x - 4$			
T: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.	A1	6
or **T**: $y = \frac{2}{3}x + \frac{5}{3}$			
T: $3y = 2x + 5$			
T: $\underline{2x - 3y + 5 = 0}$			

[13]

3.

	$x = \cos(2y + \pi)$		
	$\frac{dx}{dy} = -2\sin(2y + \pi)$		A1
	$\frac{dy}{dx} = -\frac{1}{2\sin(2y + \pi)}$	Follow through their $\frac{dx}{dy}$ before or after substitution	A1ft
At $y = \frac{\pi}{4}$,	$\frac{dx}{dy} = -\frac{1}{2\sin\frac{3\pi}{2}} = \frac{1}{2}$		B1
	$y - \frac{\pi}{4} = \frac{1}{2}x$		
	$y = \frac{1}{2}x + \frac{\pi}{4}$		A1 6

[6]

4. (a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$ M1A1

$\left(\frac{dy}{dx}\right)_0 = 6$ B1

$y - 4 = -\frac{1}{6}x$ or equivalent M1A1 5

(b) $R = \sqrt{(3^2 + 4^2)} = 5$ M1A1

		$\tan\alpha = \frac{4}{3}$, $\alpha \approx 0.927$	awrt 0.927	M1A1	4
	(c)	$\sin(2x + \text{their } \alpha) = 0$ $x = -2.03, -0.46, 1.11, 2.68$ First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.		A1A1A1	4

[13]

5.	(a)	$y = \frac{\pi}{4} \Rightarrow x = 2\sin\frac{\pi}{4} = 2\times\frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C$		B1	1
		Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or exact equivalent is used.			
	(b)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 2\cos y$ or $1 = 2\cos y\frac{\mathrm{d}y}{\mathrm{d}x}$		M1A1	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\cos y}$	May be awarded after substitution		
		$y = \frac{\pi}{4} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{2}}$ *	cso	A1	4
	(c)	$m' = -\sqrt{2}$		B1	
		$y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$		M1A1	
		$y = -\sqrt{2}x + 2 + \frac{\pi}{4}$		A1	4

[9]

6.	$\frac{dy}{dx} = \frac{1}{x}$		A1	
	At $x = 3$, gradient of normal = $\frac{-1}{\frac{1}{3}} = -3$			
	$y - \ln 1 = -3(x - 3)$			
	$y = -3x + 9$		A1	5

[5]

7. (a) $I = 3x + 2e^x$ B1

Using limits correctly to give 1 + 2e. (c.a.o.) A1 3

must subst 0 and 1 and subtract

(b) $A = (0, 5)$; B1

y = 5

$\frac{dy}{dx} = 2e^x$ B1

Equation of tangent: $y = 2x + 5$; c = –2.5 A1 4

attempting to find eq. of tangent and subst in y = 0, must be linear equation

(c) $y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \Rightarrow 4y - 2 = 5x - xy$ A1

putting y = and att. to rearrange to find x .

$g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent A1 3

must be in terms of x

(d) gf(0) = g(5); =3 A1 2

att to put 0 into f and then their answer into g

[12]

8. (a) $0 = k + \ln 2\left(\frac{1}{2e}\right) \Rightarrow 0 = k - 1 \Rightarrow k = 1$ (*) A1 2

(Allow also substituting k = 1and x = $\frac{1}{2e}$ *into equation and showing y = 0 and substituting k = 1 and y = 0 and showing x =* $\frac{1}{2e}$*.)*

(b) $\frac{dy}{dx} = \frac{1}{x}$ B1

At A gradient of tangent is $\frac{1}{1/2e} = 2e$

Equations of tangent: $y = 2e\left(x - \frac{1}{2e}\right)$

Simplifying to $y = 2ex - 1$ (*) cso A1 4

	(c)	$y_1 = 1.69$, $y_2 = 2.39$	B1, B1	2
	(d)	$\int_1^3 (1 + \ln 2x)\mathrm{d}x \approx \frac{1}{2} \times \frac{1}{2} \times (...)$	B1	
		$\approx \ldots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c)	A1ft	
		≈ 4.7	A1	4
		accept 4.67		

[12]

9.	(a)	Differentiating; $\mathbf{f'(x) = 1 + \frac{e^x}{5}}$	A1	2
	(b)	A: $\left(0, \frac{1}{5}\right)$	B1	
		Attempt at $y - \mathrm{f}(0) = \mathrm{f}'(0)x$;		
		$\boldsymbol{y - \frac{1}{5} = \frac{6}{5} x}$ or equivalent "one line" 3 termed equation	A1 ft	3
	(c)	**1.24, 1.55, 1.86**	B2(1,0)	2
	(d)	Estimate = $\frac{0.5}{2}$; (×) [(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)] B1	A1 ft	
		= **2.4275** $\left(\begin{matrix}\mathbf{2.428} \\ \mathbf{2.429}\end{matrix}, \mathbf{2.43}\right)$	A1	4

[11]

10.	(a)	(i) $\underline{x = a^y}$	B1	1
		(ii) In both sides of (i) i.e $\ln x = \ln a^y$ or (y =) $\log_a x = \frac{\ln x}{\ln a}$		
		$= \underline{y \ln a}$ * $\Rightarrow y \ln a = \ln x$	B1_{cso}	1
		B1 $x = e^{y\ln a}$ is BO		
		B1 Must see $\ln a^y$ or use of change of base formula.		

	(b)	$y = \frac{1}{\ln a}\bullet\ln x$, $\Rightarrow$ $\frac{\mathrm{d}y}{\mathrm{d}x}, = \underline{\frac{1}{\ln a}\times\frac{1}{x}}$ *	A1$_{cso}$	2
		ALT. $\left[\text{or } \frac{1}{x} = \frac{\mathrm{d}y}{\mathrm{d}x}\cdot\ln a \;\; ,\Rightarrow \;\; \underline{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x\ln a}} \; *\right]$		
		A1$_{cso}$ needs some correct attempt at differentiating.		
	(c)	$\log_{10} 10 = 1 \Rightarrow$ A is (10, $\underline{1}$) $y_A = 1$	B1	
		from(b) $\underline{m = \frac{1}{10\ln a} \text{ or } \frac{1}{10\ln 10}}$ or 0.043 (or better)	B1	
		equ of target $y - 1 = m(x - 10)$		
		i.e $\underline{y - 1 = \frac{1}{10\ln 10}(x - 10) \text{ or } y = \frac{1}{10\ln 10}x + 1 - \frac{1}{\ln 10}}$ (o.e)	A1	4
		B1 Allow either		
		ft their y_A and m		
	(d)	$y = 0$ *in* (c) $\Rightarrow 0 = \frac{x}{10\ln 10} + 1 - \frac{1}{\ln 10}$ $\Rightarrow x, = 10\ln 10\left(\frac{1}{\ln 10} - 1\right)$		
		$\underline{x = 10 - 10\ln 10}$ or $\underline{10(1 - \ln 10)}$ or $\underline{10\ln 10(\frac{1}{\ln 10} - 1)}$	A1	2
		Attempt to solve correct equation. Allow if a not = 10.		

[10]

11.	(a)	$\mathrm{f}'(x) = \frac{3}{x} - \frac{1}{x^2}$	A1	
		$\frac{3}{x} - \frac{1}{x^2} = 0 \Rightarrow 3x^2 - x = 0 \Rightarrow x = \frac{1}{3}$	A1	4
	(b)	$y = 3\ln\left(\frac{1}{3}\right) + \frac{1}{\left(\frac{1}{3}\right)} = 3 - 3\ln 3$ ($k = 3$)	A1	2
	(c)	$x = 1 \Rightarrow y = 1$	B1	
		$\mathrm{f}'(1) = 2 \Rightarrow m = -\frac{1}{2}$		
		$y - 1 = -\frac{1}{2}(x - 1)$ $\left(y = -\frac{x}{2} + \frac{3}{2}\right)$	A1	4

(d) (i) $-\frac{x}{2}+\frac{3}{2}=3\ln x+\frac{1}{x}$

leading to $6\ln x + x + \frac{2}{3} - 3 = 0$ (*) A1

cso

(ii) g(0.13) = 0.273…
g(0.14) = –0.370…

Both, accept one d.p.

Sign change (and continuity) $\Rightarrow$ root $\in$ (0.13, 0.14) A1 4

[14]

12. (a) M is (0, 7) B1

$\frac{dy}{dx} = 2e^x$

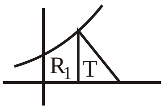
Attempt $\frac{dy}{dx}$

$\therefore$ gradient of normal is $-\frac{1}{2}$

ft their $y'(0)$ *or* $= -\frac{1}{2}$

(Must be a number)

$\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $\underline{x + 2y - 14 = 0}$
$\underline{x + 2y = 14}$ o.e. A1 4

(b)	$y = 0, x = 14 \therefore$ $\underline{N \text{ is } (14, 0)}$ (*)	B1 cso	1
(c)			
	$\int (2e^x + 5)\, dx = [2e^x + 5x]$		
	some correct $\int$		
	$R_1 = \int_0^{\ln 4} (2e^x + 5)\, dx = (2 \times 4 + 5 \ln 4) - (2 + 0)$		
	limits used		
	$= 6 + 5 \ln 4$	A1	
	$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$	B1	
	Area of T		
	$T = 13(7 - \ln 2)$; $R_1 = 6 + 10 \ln 2$	B1	
	Use of ln 4 = 2ln 2		
	$R = T + R_1$, $\underline{R = 97 - 3 \ln 2}$	A1	7

[12]

1. This question provided candidates with many opportunities to score marks, with the method of differentiation to find an equation of the normal well known. About 47% of candidates gained all 7 marks and only about 15% of candidates scored 3 or fewer marks.

Use of the chain rule to find the gradient of the curve would have been the simplest method, but this was often not the choice made by the majority of candidates. When the chain rule was used, the result was often correct with only a few sign errors being seen. The expected error of $18(5-3x)^{-1}$ was only rarely seen. The quotient rule was commonly used, but unfortunately, some candidates made errors of differentiating $u = 3$ in the numerator to give 1 or differentiating $v = (5-3x)^2$ in the denominator to give $2(5-3x)^1$. A number of candidates multiplied out their denominator to give $v = 25 - 30x + 9x^2$ before being able to differentiate to find $\frac{dv}{dx}$. Usually the quotient rule was stated correctly, or at least applied correctly with the minus sign in the formula being evident, although some candidates wrote their terms in the wrong way round and some other candidates did not square v in the denominator to give $(5-3x)^4$.

Substituting $x = 2$ to find the y-coordinate and to find the gradient of the tangent were managed well, with only a few sign errors. A few candidates found the equation of the tangent and lost the final 3 marks. Also, a number of candidates failed to write the equation of the normal in the correct form and so lost the final accuracy mark. Having said this, most candidates were able to apply $m(T).m(N) = -1$ in order to find the equation of their normal.

2. Part (i)(a) was well answered by the majority of candidates. The most common error was incorrectly differentiating $\cos 3x$ to either $3\sin 3x$ or $-\sin 3x$. A few candidates lost the final accuracy mark for simplification errors such as simplifying $(\cos 3x)(2x)$ to $\cos 6x^2$.

In (i)(b), the quotient rule was generally well applied in most candidates' working. A significant number of candidates, however, struggled to differentiate $\ln(x^2 + 1)$ correctly.

$\frac{1}{x^2+1}, \frac{2}{x^2+1}$ or even $\frac{1}{x}$ were common incorrect outcomes. Those candidates who decided to use the product rule in this part were less successful in gaining some or all of the marks.

Again part (ii) was generally well attempted by candidates of all abilities. The most common error was incorrectly differentiating $\sqrt{(4x+1)}$ although a few candidates failed to attempt to differentiate this. A few candidates found the equation of the normal and usually lost the final two marks. Also, a number of candidates failed to write the equation of the tangent in the correct form and so lost the final accuracy mark.

3. This proved a discriminating question. Those who knew what to do often gained all 6 marks with just 4 or 5 lines of working but many gained no marks at all. Although there are a number of possible approaches, the most straightforward is to find $\frac{dy}{dx}$, using the chain rule, and then invert $\frac{dy}{dx}$ to obtain $\frac{dx}{dy}$. Substituting $y = \frac{\pi}{4}$ gives the gradient of the tangent and the equation of the tangent can then be found using $y - y_1 = m(x - x_1)$ or an equivalent method. However, many confused $\frac{dy}{dx}$ with $\frac{dx}{dy}$. Those who knew the correct method often introduced the complication of expanding $\cos(2y + \pi)$ using a trigonometric addition formula. Such methods were often flawed by errors in differentiation such as $\frac{d}{dy}(\sin \pi) = \cos \pi$ Among those who chose a correct method, the most frequently seen error was differentiating cos $(2y + \pi)$ as – sin $(2y + \pi)$.

An instructive error was seen when candidates changed the variable y to the variable x while inverting, proceeding from $\frac{dx}{dy} = -2\sin(2y + \pi)$ to $\frac{dy}{dx} = -\frac{1}{2\sin(2x + \pi)}$. This probably reflected a confusion between inverting, in the sense of finding a reciprocal, and the standard method of finding an inverse function, where the variables x and y are interchanged.

4. In part (a), most candidates were aware that they needed to differentiate the given equation and this was usually done correctly although not always by the quickest method. A few did not know how to proceed from here but the majority that did, generally found the correct gradient of both the curve and the normal and subsequently a correct equation. Almost all candidates scored well on in part (b). However, many candidates lost the final mark as they gave their value of α in degrees, or their answer had come from tan $\alpha = \frac{3}{4}$.

The majority of candidates treated part (c) as a continuation of part (b), though those that noticed the route via tan $2x$ found the answer came out very easily. Most candidates found one or two solutions and, occasionally, a third solution. However obtaining all four solutions was a rare occurrence.

5. Many did gain the mark in part (a) but, again, the inappropriate use of calculators and decimals was common. The statement that 2 sin $\frac{\pi}{4}$ = √2 was not sufficient for credit and the examiners required some evidence that the candidate knew, or could show that, $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. Part (b) was well done. The great majority found $\frac{dx}{dy}$ and inverted the result. The use of implicit differentiation was rare. Apart from a few who found the equation of the tangent, part (c) was well done. The commonest cause of the loss of the final mark of the question was that candidates who had a correct form of the equation of the normal, usually $y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$, often were unable to manipulate this equation correctly into the form required.

6. **Pure Mathematics P2**

Those candidates who could differentiate the logarithmic function had no difficulty with the question. A gradient function of the form $\frac{k}{x}$ was expected. Many candidates ran into problems with the $\frac{1}{3}$, and some obtained the correct expression $\frac{\frac{1}{3}}{\frac{x}{3}}$ but did not simplify this correctly.

Most candidates obtained the correct value, $y = 0$ when $x = 3$, but some had problems with $ln1$. Almost all candidates offered a sensible attempt at the equation of the normal rather than the tangent.

Core Mathematics

The differentiation proved difficult and both $\frac{3}{x}$ and $\frac{1}{3x}$ were common. Even if a correct expression for $\frac{dy}{dx}$ was found, difficulties with simplifying fractions often led to incorrect work. For example, $\frac{\frac{1}{3}}{\frac{1}{3}x}$ was sometimes seen simplified to x. Failure to read the question carefully lead a number of candidates to give the tangent rather than the normal. There were also candidates who did not find a numerical gradient and gave a non-linear equation for the normal.

7. This question was relatively well answered. In part (a) most candidates integrated correctly though some candidates did not use the limit $x = 0$. In part (b) a minority of candidates left the gradient of the tangent as $2e^{2x}$. A few thought the gradient of the tangent was -1/2. In part (c) the majority of candidates knew what to do but poor algebraic skills gave incorrect answers Part (d) was usually well done, though some found $g^{-1}f(0)$ and the inevitable few fg(0).

8. In part (a), the log working was often unclear and part (b) also gave many difficulty. The differentiation was often incorrect. $\frac{1}{2x}$ was not unexpected but expressions like $x + \frac{1}{x}$ were also seen. Many then failed to substitute $x = \frac{1}{2e}$ into their $\frac{dy}{dx}$ and produced a non-linear tangent. Parts (c) and (d) were well done. A few did, however, give their answers to an inappropriate accuracy. As the table is given to 2 decimal places, the answer should not be given to a greater accuracy.

9. For many candidates this was a good source of marks. Even weaker candidates often scored well in parts (c) and (d). In part (a) there were still some candidates who were confused by the notation, f' often interpreted as f^{-1}, and common wrong answers to the differentiation were $\frac{e^x}{5}$ and $1 + e^x$. The most serious error, which occurred far too frequently, in part (b) was to have a variable gradient, so that equations such as $y - \frac{1}{5} = \left(1 + \frac{e^x}{5}\right)x$ were common. The normal, rather than the tangent, was also a common offering.

10. Part (a) (i) was usually answered well but the provision of the answers in the next two parts meant that a number of candidates failed to score full marks either through failing to show sufficient working, or by the inclusion of an incorrect step or statement.

The most successful approach to part (b) started from $y = \frac{\ln x}{\ln a}$ but there was sometimes poor use of logs such as $\frac{\ln x}{\ln a} = \ln x - \ln a = \ln\left(\frac{x}{a}\right)$; incorrect notation such as $\frac{dy}{dx}\ln x = \frac{1}{x}$,or errors in differentiation when $\frac{1}{a}$ appeared.

In part (c) the process for finding an equation for the tangent was usually well known but some candidates did not appreciate that the gradient should be a constant and gave a non-linear equation for their tangent. Some confused ln10 with $\log_{10}10$. Many candidates had problems working exactly in the final part and others ignored this instruction and gave an answer of –13.02.

11. Part (a) was usually well done by those who knew what a stationary value was although minor algebraic errors often spoilt essentially correct solutions. A few candidates either had no idea what a stationary value was, and began the question at part (c), or thought that they needed the second, rather than the first, derivative equal to zero. Part (b) proved the hardest part of this question and many were not able to use a correct law of logarithms to find *k*. Part (c) was very well done but part d(i) produced some rather uncertain work. Many candidates did not seem to realise what was expected. Nearly all candidates knew the technique required in part (d)(ii), although a few substituted 0.13 and 0.14 into f(x). The examiners do require a reason and a conclusion at the end of such a question.

12. Whilst the majority of answers to part (a) were fully correct, some candidates found difficulties here. A small number failed to find the coordinates of *M* correctly with (0, 5) being a common mistake. Others knew the rule for perpendicular gradients but did not appreciate that the gradient of a normal must be numerical. A few students did not show clearly that the gradient of the curve at $x = 0$ was found from the derivative, they seemed to treat $y = 2e^x + 5$ and assumed the gradient was always 2. Some candidates failed to obtain the final mark in this section because they did not observe the instruction that *a, b* and *c* must be integers.

For most candidates part (b) followed directly from their normal equation. It was disappointing that those who had made errors in part (a) did not use the absence of $n = 14$ here as a pointer to check their working in the previous part. Most preferred to invent all sorts of spurious reasons to justify the statement.

Many candidates set out a correct strategy for finding the area in part (c). The integration of the curve was usually correct but some simply ignored the lower limit of 0. Those who used the simple “half base times height” formula for the area of the triangle, and resisted the lure of their calculator, were usually able to complete the question. Some tried to find the equation of *PN* and integrate this but they usually made no further progress. The demand for exact answers proved more of a challenge here than in 6(c) but many candidates saw clearly how to simplify $2e^{\ln 4}$ and convert ln 4 into 2 ln 2 on their way to presenting a fully correct solution.